Corrigendum to "On the zeros of polynomials: An extension of the Eneström-Kakeya theorem"

Vanessa Botta*

Faculdade de Ciências e Tecnologia, UNESP - Univ Estadual Paulista, Departamento de Matemática, Estatística e Computação, P. O. Box 467, 19060-900 Presidente Prudente, SP, Brazil

Messias Meneguette

Faculdade de Ciências e Tecnologia, UNESP - Univ Estadual Paulista, Departamento de Matemática, Estatística e Computação, P. O. Box 467, 19060-900 Presidente Prudente, SP, Brazil

José A. Cuminato

Departamento de Matemática Aplicada e Estatística, Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo, P. O. Box 668, 13560-970 São Carlos, SP, Brazil

Sean McKee

Department of Mathematics and Statistics, University of Strathclyde, 26 Richmond Street, Glasgow G1 1XH, Scotland, U. K.

Abstract

In the paper [1] we presented an extension of the Eneström-Kakeya theorem concerning the roots of a polynomial that arises from the analysis of the stability of Brown (K, L) methods. We note that in this paper an index is missing from Lemma 9 and the word "inside" has been typed instead of "outside" just after Lemma 10. These are very minor points and have no effect on the main results of this article.

*Corresponding author

Email addresses: botta@fct.unesp.br (Vanessa Botta), messias@fct.unesp.br (Messias Meneguette), jacumina@icmc.usp.br (José A. Cuminato), s.mckee@strath.ac.uk (Sean McKee)

Preprint submitted to Journal of Mathematical Analysis and Applications December 2, 2011

Given $P(z) = \sum_{i=0}^{n} a_i z^i$, $a_i \in \mathbb{R}$, the sequence of polynomials $P_j(z)$ is defined by

$$P_j(z) = \sum_{k=0}^{n-j} a_k^{(j)} z^k, \quad \text{where} \quad P_0(z) = P(z) \quad \text{and}$$

$$P_{j+1}(z) = \Delta P_j(z) \doteq a_0^{(j)} P_j(z) - a_{n-j}^{(j)} P_j^*(z), \; j = 0, 1, \ldots, n-1,$$

with $P_0^*(z) = P^*(z) = z^n P\left(\frac{1}{z}\right)$ and $P_j^*(z) = (P_j(z))^*$.

Observe that the polynomial $P(z) = z^3 - 4z^2 + 5z - 2$ has two zeros on the unit circle and $P_2(z) \equiv 0$. This case represents a counter-example to Lemma 9 from [1]. So, the statement and the proof of Lemma 9 from [1] must be written as follow.

Lemma 1. *Let $P(z)$ be a polynomial with real coefficients. Suppose $P(z)$ has q zeros on the unit circle then $P_{n-q+1}(z) \equiv 0$. In particular if $P(z)$ has all its roots on the unit circle then $\Delta P(z) \equiv 0$.*

Proof. From the last statement of Lemma 8 in [1], $P_{n-q+1}(z)$ has the same roots as $P_{n-q}(z)$ on the unit circle. By a recursive argument this polynomial has the same zeros as $P_{n-q-1}(z)$ on the unit circle and so on, leading to the conclusion that it has the same zeros as $P(z)$ on the unit circle. In conclusion $P_{n-q+1}(z)$ has q roots on the unit circle, but $P_{n-q+1}(z)$ is a polynomial of degree less than or equal $q-1$ by construction and hence it must vanish. $\square$

Furthermore, the correct way to write the statement after Lemma 10 from [1] is "If $|a_0| > |a_n|$ and $P(z)$ has all zeros outside the unit circle, from Lemma 8 in [1], $\Delta P(z)$ has all zeros outside the unit disk and, consequently, $\Delta P^*(z)$ has all zeros inside the unit circle".

Acknowledgements

These points were brought to our attention by Mr Heydar Ali Soleiman Mezerji.

References

[1] V. Botta et al., *On the zeros of polynomials: an extension of the Eneström-Kakeya theorem*, J. Math. Anal. Appl. (2011), doi 10.1016/j.jmaa.2011.07.037